Published by Institute of Physics Publishing for SISSA/ISAS
Received: *October 26, 2003*
Accepted: *November 7, 2003*

Adding a Myers term to the IIB matrix model

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Abstract: We show that Yang-Mills matrix integrals remain convergent when a Myers term is added, and stay in the same topological class as the original model. It is possible to add a supersymmetric Myers term and this leaves the partition function invariant.

Keywords: Nonperturbative Effects, M(atrix) Theories, Matrix Models.

JHEP11(2003)009

Contents

1. Introduction

The original motivation of [1] for studying reduced Yang-Mills matrix models with added cubic terms was to understand static $D0$ brane solutions in a constant RR background field. It was shown that separated branes condense into a noncommutative space configuration, and this is known as the Myers effect.

It is also interesting to add a Myers term to the action of completely reduced Yang-Mills matrix models. This gives an action which realises gauge theory on a noncommutative space [2, 3], and would therefore be an interesting deformation of the IIB matrix model [4]. In this article, we will make heavy use of the existing analytic methods for arbitrary gauge groups [5]–[9] to show that adding the Myers term *is* a legitimate deformation of the original matrix model.

As the author of [3] points out, the Myers term formally vanishes in the large-N limit. This means that it may act as a useful order parameter to study a possible dynamical reduction of the matrix model [10] onto lower dimensional spaces [2, 11, 12]. The possibility of such dynamical dimensional reduction of the model has been investigated since early on [10]–[24], and for a recent review see [25]. In addition, one can consistently preserve half of the supersymmetries of the matrix model while adding Myers type terms, and this is one step in the MNS method [26] for computing the SYMM partition function (see also [27, 28, 29, 30, 31, 8, 32]).

Addition of the Myers term takes the form

$$S_{\mathrm{YM}} \to S_{\mathrm{YM}} + \frac{i}{3}\lambda\, f_{\mu\nu\rho}\,\mathrm{Tr}\,[X_\mu, X_\nu]\,X_\rho \tag{1.1}$$

where $f_{\mu\nu\rho}$ is an antisymmetric tensor. If we assume that λ is a real parameter, then we are led to a puzzle. We have added unbounded cubic terms to the action, and so one would guess that the partition function should diverge. In the simplest example of a bosonic action with added Myers term

$$S = \mathrm{Tr}\,[X_\mu, X_\nu]\,[X_\mu, X_\nu]^\dagger + i\lambda\,\mathrm{Tr}\,X_1\,[X_2, X_3]\,, \tag{1.2}$$

we can imagine fixing $[X_2, X_3]$ and taking X_1 to ∞ in such a way that the action would be unbounded from below. On the other hand, we could rewrite the trace as

$$i\,\mathrm{Tr}\,[X_1, X_2]\,X_3\,. \tag{1.3}$$

Now it is the commutator $[X_1, X_2]$ which goes to ∞, and this can be overcome by the quadratic appearance of the commutator in the Yang-Mills part of the action. However, it could be that while $[X_2, X_3]$ itself was fixed, X_3 is going to ∞. The main purpose of this paper is to show that no matter how we take the matrices X_1, X_2 and X_3 to ∞, the large negative contribution to the action is always overcome in time to save convergence of the partition function.

Although it has been known for some time how to compute the $su(2)$ supersymmetric pure partition functions (i.e. without Myers term) exactly [33]–[38], it was not realised until [39] that the bosonic integrals can also converge. It was originally believed that integrating along the flat directions in which the matrices commute would lead to divergence, while the pfaffian in the supersymmetric case could save the situation. However, the authors of [39] calculated the $su(2)$ bosonic partition functions exactly, and found that they converge for $D \geq 5$ and using careful numerical methods, found convergence for various other low rank gauge groups [39, 27, 28, 29, 30, 31]. In [6] we gave analytic convergence criteria for the $su(N)$ bosonic models, and extended these to the other gauge groups and supersymmetric models in [7]. These methods have also been used to study the large momentum behaviour of Polyakov lines [9], and we use them again here to obtain the results of this work.

In a recent article [40], D. Tomino obtains a finite result for the $D = 3$ $su(2)$ partition function with Myers term. Our analysis does not apply to this case since, while the integral without Myers term is formally zero, it is absolutely divergent. As we shall see, even when the Myers term is large, it is always small compared to the bosonic part of the action. This seems to allow the same cancellations which saved the pure partition function to also save the version with Myers term in this case.

In this note, we show that in fact the partition function with added Myers term *is* convergent as long as the pure $\lambda = 0$ partition function converges absolutely. We also show that adding a Myers term gives a true deformation of the pure Yang-Mills model and does not lead to a different topological class of models. This is clearly important if we are to use it as a probe of symmetry breaking in the pure model, and is not obvious; the supersymmetric mass terms added in the MNS method for example lead to a model which diverges when the masses go to zero [26, 8]. Finally, we write down supersymmetric Myers terms and show that, in this case, the partition function takes the same value as the $\lambda = 0$ version, that is it is independent of λ.

2. Dealing with a Myers term

We consider integrals of the form

$$\mathcal{Z}(\lambda) = \int \prod_{\mu=1}^{D} dX_\mu\, \mathcal{P}(X_\mu) \exp\left(-S_D(X) + i\lambda\epsilon_{\mu\nu\rho}\,\mathrm{Tr}\,[X_\mu, X_\nu]\,X_\rho\right) \tag{2.1}$$

where $S_D(X) = \mathrm{Tr}\,[X_\mu, X_\nu]\,[X_\mu, X_\nu]^\dagger$ is the bosonic action and $\mathcal{P}(X)$ is a pfaffian generated by integrating out any fermions. We cover the supersymmetric cases, and also the bosonic case in which the pfaffian is absent. We use the standard properties of Lie algebras which apply to Yang-Mills integrals and are described in for example [7].

The problem is that terms of the form $i\lambda\,\mathrm{Tr}\,[X_1, X_2]\,X_3$ are real and can be large and positive as well as negative. To deal with this, we show that

$$|i\,\mathrm{Tr}\,[X_1, X_2]\,X_3| \leq C_1 \left|\mathrm{Tr}\,[X_1, X_2]^2 + [X_2, X_3]^2 + [X_3, X_1]^2\right| + C_2 \tag{2.2}$$

that is, that the Myers term can be bounded by the bosonic part of the action plus a constant.

Lets suppose that

$$|X_1| \geq |X_2|, |X_3| \tag{2.3}$$

and restrict to the region in which $|X_1| > 1$. We can diagonalise X_1

$$X_1 \to \underline{x} \cdot H, \tag{2.4}$$

where H^i are the Cartan generators. It is convenient to use a basis in which $x_i = \underline{x} \cdot s_i$, where $\{s_i\}$ are the simple roots. Then by a Weyl transformation, we can assume [7]

$$x_1, \ldots, x_r \geq 0 \tag{2.5}$$

$$x_1 \geq x_2 \geq \ldots \geq x_r\,. \tag{2.6}$$

That is, first we choose our definition of positivity of roots so that 2.5 is true, and then we relabel the simple roots so that 2.6 is true.

Then we can rewrite the Myers term as

$$i\lambda\,\mathrm{Tr}\,X_1\,[X_2, X_3] = \lambda \sum_\alpha (\underline{x} \cdot \alpha) X_2^\alpha X_3^{-\alpha} \tag{2.7}$$

where the sum is over all roots. Recall that every root can be written $\alpha = \pm \sum_{i=1}^r n_i s_i$ where the n_i are nonnegative integers. Then we look at just one of the terms, α, in the sum, for which the root α contains s_1. Then since $|X_1| \geq 1$ and since x_1 is the biggest x_i, we have

$$\left|(\underline{x} \cdot \alpha) X_2^\alpha X_3^{-\alpha}\right| \leq \mathrm{const}(\underline{x} \cdot \alpha)^2 |X_2^\alpha|^2 \tag{2.8}$$

where, without loss of generality, we have assumed that $|X_2^\alpha| \geq |X_3^\alpha|$.

The rhs of 2.8 is just one term in the expansion $\left|\mathrm{Tr}\,[X_1, X_2]^2\right| = \sum_\alpha (\underline{x} \cdot \alpha)^2 |X_2^\alpha|^2$ and so we have the bound

$$\left|\underline{x} \cdot \alpha X_2^\alpha X_3^{-\alpha}\right| \leq \mathrm{const}\left|\mathrm{Tr}\,[X_1, X_2]^2\right| + \mathrm{const} \tag{2.9}$$

whenever α contains s_1. Here we have included the additive constant to take care of the case in which $|X_1| \not> 1$.

At this stage, we have dealt with all the terms α containing s_1, and so we have reduced the problem to that for the regularly embedded subalgebra obtained by removing the simple root s_1. Then, by induction, we have the result 2.2.

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As usual, our strategy is to prove absolute convergence of the partition function. So far, we have the bound

$$|\exp\left(-S_D(X) + i\epsilon_{\mu\nu\rho}\,\mathrm{Tr}\,[X_\mu, X_\nu]\,X_\rho\right)| \leq \exp\left(-S_D(X) + C_1 S_D(X) + C_2\right). \tag{2.10}$$

The right hand side would only give a convergent integral if $C_1 < 1$. However, the divergence in the integrals comes from the region in which the radial variable $R = \sqrt{\mathrm{Tr}\,X_\mu X_\mu}$ goes to infinity. Since the Myers term is cubic, while the bosonic part of the action, S_D is quartic, we can choose C_1 to be as small as we like by looking at a region with R large enough. We deduce that the partition function

$$\mathcal{Z}(\lambda) = \int \prod_{\mu=1}^{D} dX_\mu\, \mathcal{P}(X_\mu) \exp\left(-S_D(X) + i\lambda\epsilon_{\mu\nu\rho}\,\mathrm{Tr}\,[X_\mu, X_\nu]\,X_\rho\right) \tag{2.11}$$

is convergent for any λ, as long as the pure Yang-Mills integral $\mathcal{Z}(0)$ is absolutely convergent.

It would be interesting to know whether the Myers term gives a continuous deformation of the Yang-Mills integral, or whether it gives a different toplogical class of model. In other words, we would like to know whether the limit of $\mathcal{Z}(\lambda)$ is $\mathcal{Z}(0)$ when $\lambda \to 0$. We can now answer this very simply. By Taylor's theorem,

$$\mathcal{Z}(\lambda) - \mathcal{Z}(0) = \lambda \int \prod_{\mu=1}^{D} dX_\mu\, i\,\mathrm{Tr}\left([X_1, X_2]\,X_3\right)\, \mathcal{P}(X_\mu) \exp\left(-S_D(X) + i\hat{\lambda}\,\mathrm{Tr}\,[X_1, X_2]\,X_3\right) \tag{2.12}$$

for some $|\hat{\lambda}| < |\lambda|$. Here we have chosen a particular Myers term without losing any generality. We just need to check that the integrals in the rhs are convergent, and as usual, the only possible region of divergence is at large R. At large R, we can bound the rhs by

$$\lambda \int \prod_{\mu=1}^{D} dX_\mu\, S_D(X)\,|\mathcal{P}(X)| \exp\left(-S_D(X) + \hat{\lambda} S_D(X)\right) \tag{2.13}$$

up to a constant factor. Finally, we need to recall some details from the proofs of convergence [6, 7]. We can restrict integration to the region in which $S_D(X) < R^\epsilon$ where ϵ is an arbitrarily small positive constant, since outside this region the integrand decays exponentially. If we integrate out the angular variables we obtain a bound

$$\int_1^\infty \frac{dR}{R} R^\epsilon R^{-k_c(D)+\epsilon'} \tag{2.14}$$

where the k_c are numbers, depending on the algebra as well as the dimension D, which are calculated explicitly in [7], and ϵ' is another arbitrarily small, positive constant. When the pure ($\lambda = 0$) matrix model is convergent, $k_c(D)$ must be positive, and so choosing ϵ, ϵ' small enough we find that 2.13 is indeed convergent. Finally then, we have

$$|\mathcal{Z}(\lambda) - \mathcal{Z}(0)| < \mathrm{const}\,\lambda \to 0 \tag{2.15}$$

as $\lambda \to 0$.

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3. Supersymmetric Myers term

It is possible to add a Myers term to the supersymmetric models and still preserve half of the supersymmetry [26, 8]. For example, for $D = 4$, we can write the action

$$S = \mathrm{Tr}\left(\left(H+\frac{1}{2}[X_1, X_2]\right)^2 - \frac{1}{4}\sum_{\mu>\nu}[X_\mu, X_\nu]^2 + i\lambda\,\mathrm{Tr}(-X_3+iX_4)[X_1, X_2] - \right. \tag{3.1}$$

$$\left. - \epsilon_{ab}\eta_1[\psi_a, X_b] - \eta_a\frac{1}{2}[(X_3+iX_4), \eta_a] - \psi_a\frac{1}{2}[(-X_3+iX_4), \psi_a] + \eta_2[\psi_a, X_a]\right).$$

where H is an auxilliary matrix, and the η_a and ψ_a, $a = 1, 2$ are the fermions. We have added the Myers term $i\lambda\,\mathrm{Tr}(-X_3 + iX_4)[X_1, X_2]$.

The action is δ-exact, $S = \delta\,\mathrm{Tr}\,Q$, where

$$Q = \left(\eta_1[X_1, X_2] + \eta_1 H + \frac{1}{2}\psi_a[X_a, X_3 - iX_4] - \frac{i}{2}\eta_2[X_3, X_4]\right) \tag{3.2}$$

and

$$\begin{aligned} \delta X_a &= \psi_a\,, & \delta\psi_a &= \frac{1}{2}[X_3 + iX_4, X_a] + i\lambda\epsilon_{ab}X_b \\ \delta X_3 &= \eta_2\,, & \delta\eta_2 &= \frac{i}{2}[X_4, X_3] \\ \delta\eta_1 &= H\,, & \delta H &= \frac{1}{2}[X_3 + iX_4, \eta_1] \\ \delta X_4 &= i\eta_2 \end{aligned} \tag{3.3}$$

and since δ squares to a gauge transformation plus a rotation, one can quickly see that $\delta S = 0$. The same construction can also be made in $D = 6$ and $D = 10$ [26], and one can check that, in $D = 4$ for example, two of the four supercharges can be preserved in this way [8]. The supercharge 3.3 represents a deformation of the pure ($\lambda = 0$) version. It is not possible to generate a pure Myers term (ie without additional fermion masses) without deforming the supercharge. One can quickly see this since it would have to take the form $\delta\,\mathrm{Tr}\,P$ with P a polynomial in the matrices [5].

The results of the previous section show us that the partition function

$$\mathcal{Z}(\lambda) = \int \exp(-S) \tag{3.4}$$

is convergent, and that $\mathcal{Z}(\lambda) \to \mathcal{Z}(0)$ as $\lambda \to 0$. With the above technology, one can now see that $\mathcal{Z}$ is independent of λ. This is a property of rotation invariance rather than supersymmetry, but it will only work for a Myers term of the given form which *is* imposed by supersymmetry.

First, since $\partial\mathcal{Z}/\partial\lambda$ exists and is continuous for all λ, and $\frac{\partial\mathcal{Z}}{\partial\bar{\lambda}} = 0$, $\mathcal{Z}(\lambda)$ is analytic. But, by a 2D rotation of the X_3, X_4 matrices, we have also[1]

$$\mathcal{Z}(\lambda) = \mathcal{Z}(e^{i\theta}\lambda)\,. \tag{3.5}$$

That is, $\mathcal{Z}$ depends only on the magnitude of λ and is therefore independent of λ.

[1] We are grateful to M. Staudacher for originally pointing out this kind of phase argument.

Acknowledgments

We are grateful to Graziano Vernizzi for valuable discussion, and PA acknowledges an EPSRC fellowship.

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